CubeSim

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Eksperter i Team — NTNU 2002

"Our goal is to develop a tool that calculates the energy available to the satellite, given different parameters".

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Summary

To increase the knowledge and understanding for teamwork, all fourth year students at NTNU take the course, "Eksperter i Team".

Our group worked on the Norwegian student satellite project, *ncube*, which goal is to launch a Norwegian satellite within the year 2003. Six groups have studied different aspects of building and designing a nano satellite. Our task was to find out how much electrical energy the satellite can generate by using solar panels. To do this we designed a simulation tool *CubeSim*, that calculates the generated power as a funtion of time and satellite position.

By simulating different scenarios an estimate of a mean power production of 2.4-3.5 Watt was found to be feasible.

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Chapter 1

Introduction

In 2001 the administrative department of NTNU decided to introduce a new course, "Eksperter i Team" which is compulsory to all fourth year students. This course gather all fourth year students and mix them in small groups and assign each group a different problem.

The purpose of the course is to learn students how to work in a team. How different strategies and methods can be used, to get a team of people to work together and solve a common problem. Among many different tasks proposed by the university, 28 students chose the "ncube - norwegian student satellite" project. The students where divided into six groups, and each group assigned a different problem.

1.1 The ncube satellite project

The *ncube* satellite project is a collaboration between Andøya Rocketrange (ARS), Norwegian Space centre (NRS), Nasjonalt Senter for Romrelatert Opplæring (NAROM) and students from the University College of Narvik (HiN), Agricultural University of Norway (NLH) and Norwegian Technical University (NTNU). The ultimate goal is to launch a nano satellite, planned and built by norwegian students, within the year 2003. The satellite is a cube with the dimension 10x10x10cm.

1.2 Task spesifiation

A small satellite like ncube has limited physical area extension and the Sun is the main energy source. The satellite's ability to produce and store electrical power is thus a very important subject. To design a satellite that can convert as much solar energy into electrical power as possible is a great challenge. Therefore our team chose to look closer into the problem of how much energy a small satellite can generate and to design a tool for estimation.

1.3 Outline of the Report

An introduction to orbital mechanics, reference frames and the satellite's environment is given in Chapter 1. In Chapter 2 a brief discussion of the satellite's possible orbits is given. Chapter 3 covers energy calculation theory. Chapter 4 briefly discuss the implementation of the CubeSim simualtor. In Chapter 5 simulations of different scenarios are presented and discussed. Chapter

6 summarize the results and give recommendations for further work. A short user's guide to the CubeSim simulator is found in appendix A.

Chapter 2

Orbital Mechanics

Johannes Kepler(1571-1630) formulated the three famous laws of planetary motion from an empirical study based on data collected by the astronomer *Tycho Brahe*(1546-1601). Kepler's laws describe the simplest form of motion of celestial bodies under the assumption that no external perturbing forces are present, and that the respective masses can be considered point masses.

The three laws gave a description of the motion but no explanation. Kepler himself was convinced that his empirically found laws followed a more general law. In 1687 *Isaac Newton*(1642-1727) published his three laws of motion and the law of universal gravitation in the *Mathematical Principles of Natural Philosophy.* By using these laws Kepler's laws can be derived.

2.1 Reference frames

To describe and analyze the motion of a satellite, it is necessary to define some reference frames relative to the motion. A smart choice of reference system or coordinate system can greatly simplify the equations of motion. The following reference frames are used in this report.

Earth Centered Inertial frame ECI The Earth-centered inertial frame (ECI) is an inertial frame in which Newton's laws of motion apply. The z_i-axis points towards the geographic north pole while the x_i-axis points towards the *vernal equinox.* This direction can be found by drawing a line from Earth to the Sun on the first day of spring. The y_i-axis is found by using the right-hand rule.

Another name used for this reference frame is the *geocentric-equatorial coordinate system.* Note that the ECI frame is not a true non-accelerating reference frame, but the approximation is good enough for our purposes.

Orbit frame The orbit reference frame is located in the mass center of the satellite. The negative z_o-axis points towards the center of the Earth, while the x_o-axis points forward in the travelling direction.

Body frame The body frame is moving and rotating with the satellite. The point of origin is the same as in the orbit frame. Rotation about the axis x_b, y_b and z_b is defined as *roll*, *pitch* and *yaw* respectively.

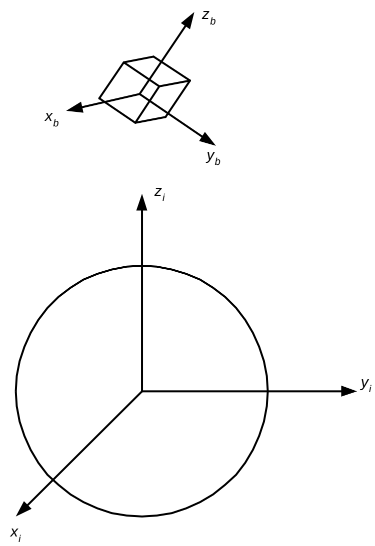


Figure 2.1: ECI and body frames

2.1.1 Transformation between ECI and Orbit frames

We assume that the satellite has an attitude control system that aligns the z-axis of the body frame with the z-axis in the orbit frame, $z_b = z_o$. The position of the satellite is given by the vector $\mathbf{r}^I$ in the ECI frame. To transform from the ECI frame to the orbit frame we therefore have to align the z_o-axis with the $\mathbf{r}^I$ vector. This rotation can be done by using the rotation matrix(Egeland, 2001)

$$\mathbf{R}_I^O = \mathbf{R}_{y_i,-\epsilon}\mathbf{R}_{z,\lambda} \tag{2.1}$$

where

$$\mathbf{R}_{y,\epsilon} = \begin{bmatrix} \cos(\epsilon) & 0 & \sin(\epsilon) \\ 0 & 1 & 0 \\ -\sin(\epsilon) & 0 & cos(\epsilon) \end{bmatrix} \quad \mathbf{R}_{z,\lambda} = \begin{bmatrix} \cos(\lambda) & -\sin(\lambda & 0) \\ \sin(\lambda) & \cos(\lambda) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.2}$$

Vectors in the ECI-frame can then be transformed to the body-frame using

$$\mathbf{v}^O = \mathbf{R}_I^O \mathbf{v}^I. \tag{2.3}$$

2.2 The Satellite Environment

2.2.1 The Earth

The Earth is often assumed to be a sphere. In reality the Earth has an equatorial bulge. The Earths's radius is about 22km larger along the equator than through the poles. For a more accurate description of the Earth's shape a oblate spheroid can be used, but a spherical model is adequate for our purposes.

2.2.2 The Sun

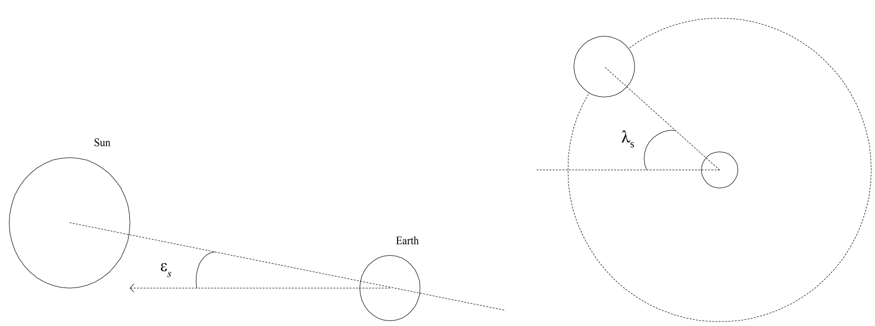
The Earth evolves around the Sun in an near circular orbit. The elevation ϵ_e of the Sun varies approximately $\pm 23°$ over a year. At the first day of spring the elevation is $\epsilon_s = 0$. Then it increase from $0°$ to $23°$ before it decreases to $-23°$ and back again to $0°$. Over a period of 365 days(one year) the elevation can approximately be described by:

$$\epsilon_s = \frac{23\pi}{180} \sin\left(\frac{T_s}{365} 2\pi\right) \tag{2.4}$$

where T_s is the number of days elapsed since the first day of spring. For simplicity we can imagine that the Sun evolves around the Earth. The azimuthal component λ_s can then be determined by:

$$\lambda_s = \frac{T_s}{365} 2\pi \tag{2.5}$$

The Sun elevation and azimuth is illustrated in figure 2.2(a) and figure 2.2(b) respectively.



(a) Sun elevation

(b) Sun azimuth in an imagined orbit around the Earth

Figure 2.2: Sun azimuth and elevation

The shortest distance from the Earth to the Sun is approximately $r_s = 1.48 \cdot 10^8$ km. By comparison the distance from the center of the Earth to the satellite is approximately $r_o = 6.98 \cdot 10^3$ km. Imagine a scenario where the Sun elevation is $\epsilon_s = 0$ and the satellite is directly over the north pole of the Earth. The angle α in figure 2.3 is then

$$\alpha = \tan^{-1}\left(\frac{r_o}{r_s}\right) \approx 4.8 \cdot 10^{-5} \, [rad] \tag{2.6}$$

which is negligible. The sunlight hitting the satellite can therefore be consider the same as if the satellite was placed in the center of the Earth.

2.2.3 The Earth's Shadow

To determine if the satellite is in the Earth's shadow or not, a simple cylindric shadow-model can be used (Seeber, 1993). The principle is shown in figure 2.4. The satellite is in sunlight

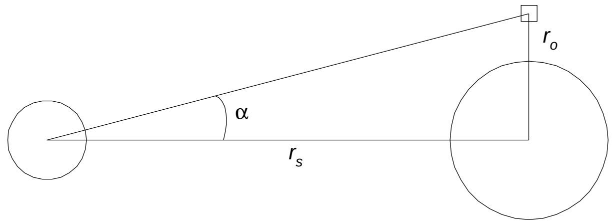


Figure 2.3: Scenario with the satellite directly over the north pole.

when

$$D = \mathbf{r}^T \mathbf{r}_s > 0 \tag{2.7}$$

and in shadow when

$$\begin{aligned} D &< 0 \text{ and} \\ |\mathbf{S}_c| &= |\mathbf{r} - D\mathbf{r}_s| < a_e. \end{aligned} \tag{2.8}$$

$\mathbf{r}_s$ is the unit vector to the Sun, and a_e is the semi-axis of the shadow generating body. By using equation (2.4) and (2.5) $\mathbf{r}_s$ is found to be:

$$\mathbf{r}_s^T = \frac{\begin{bmatrix} \cos(\lambda_s) & \sin(\lambda_s) & \epsilon_s \end{bmatrix}}{\sqrt{\cos^2(\lambda_s) + \sin^2(\lambda_s) + \epsilon_s^2}} \tag{2.9}$$

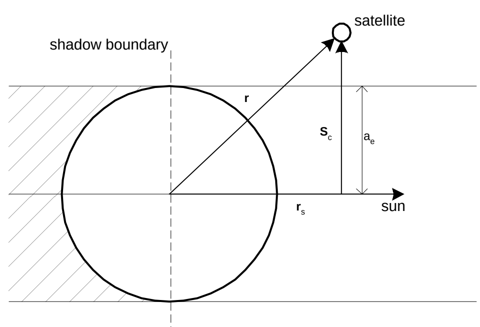


Figure 2.4: Cylindric shadow model

2.3 The Two-Body problem

In celestial mechanics we are concerned with motions of celestial bodies under the influence of mutual mass attraction. This Keplerian motion is described by 2.10.

$$\ddot{\mathbf{r}} = -G\frac{M+m}{r^3}\mathbf{r} \tag{2.10}$$

For a artificial Earth satellite the mass m can be neglected. The expression above then becomes

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r} \tag{2.11}$$

with $\mathbf{r}$ being the geocentric position of the satellite.

Equation (2.11) is a second order differential equation with solution on the form

$$\begin{aligned} \mathbf{r}(t) &= \mathbf{r}(t; a_1 \cdots a_6) \\ \dot{\mathbf{r}}(t) &= \dot{\mathbf{r}}(t; a_1 \cdots a_6) \end{aligned} \tag{2.12}$$

with $a_1 \cdots a_6$ being free selectable integration constants describing the orbit. Usually the six Keplerian orbital parameters $a, e, i, \Omega, \omega, \nu$ are used.

2.4 Classical Orbital Elements COE

To describe a sattelite orbit the six Keplerian elements $a, e, i, \Omega, \omega, \nu$ can be used. See table 2.1. For a good and pedagogic description of the classical orbital elements see Sellers (2000). The COEs are best understood by looking at figure 2.5.

Table 2.1: The six classical orbital elements

Name	**Symbol**
Semimajor axis	a
Eccentricity	e
Inclination	i
Longitude of ascending node	Ω
Argument of perigee	ω
True anomaly	ν

Semimajor axis a The major axis of an elliptical orbit is the distance between the point of closest approach (perigee) and furthest point (apogee). Semimajor axis is one-half this distance.

Eccentricity e A circular orbit has an eccentricity of zero. Elliptical orbits has an eccentricity less than one, while hyperbolic orbits has an eccentricity greater than one.

Inclination i Describes the tilt of the orbital plane with the respect to the equatorial plane.

Longitude of ascending node Ω Also called *right ascension of the ascending node*. The ascending node is the point where the satellite crosses the equator moving south to north.

Argument of perigee ω Location of the perigee with respect to the ascending node.

True anomaly ν Location of satellite with respect to perigee

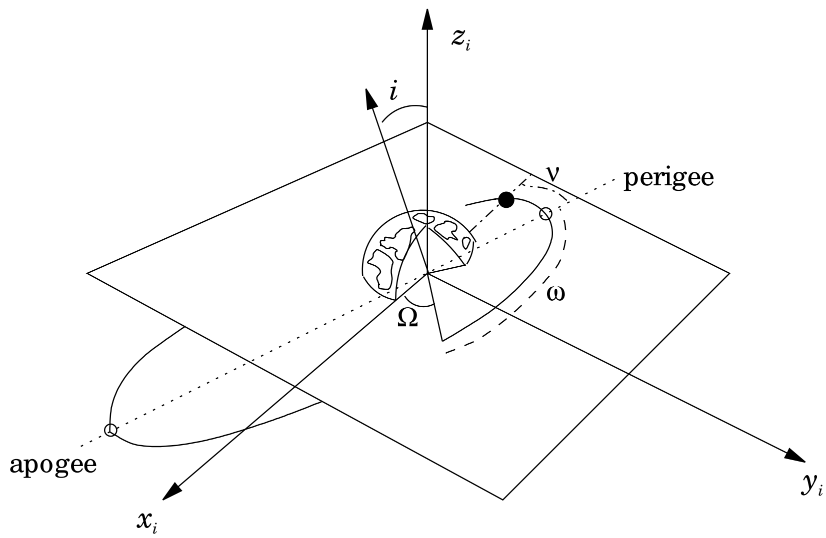


Figure 2.5: Orbital elements

Mean anomaly M

Instead of the true anomaly ν the mean anomaly M can be used. M is defined by

$$M = E - e \sin E \tag{2.13}$$

where E is given by

$$\cos E = \frac{e + \cos \nu}{1 + e \cos \nu} \tag{2.14}$$

2.4.1 The orbital period

The orbital period, P is the time it takes for the satellite to revolve once around its orbit. The period can be derived from Kepler's Third Law(Sellers, 2001) as

$$P = 2\pi \sqrt{\frac{a^3}{\mu}} \tag{2.15}$$

where P is the period in seconds, a is the semimajor axis, while $\mu = 3986005 \cdot 10^8 \frac{m^3}{s^2}$ is the gravitational parameter. For a satellite in a circular orbit 600km above the Earth's surface P is approximately 97 minutes.

2.4.2 Conversion from COEs to the ECI frame

A satellite position given in COEs can be converted to the ECI frame by(Seeber 1993)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = r \cdot \begin{bmatrix} \cos(\nu + \omega) \cos \Omega - \sin(\nu + \omega) \sin \Omega \cos i \\ \cos(\nu + \omega) \sin \Omega + \sin(\nu + \omega) \cos \Omega \cos i \\ \sin(\nu + \omega) \cos i \end{bmatrix} \tag{2.16}$$

where r is given by

$$r = \frac{a(1+e^2)}{1+e\cos\nu} \tag{2.17}$$

2.5 Perturbed Satellite Motion

Equation (2.11) describes the ideal motion of a satellite around a body when only the mutual gravitational forces are considered. In reality a certain number of additional forces act on a satellite. The extended equation of motion is given by:

$$\ddot{\mathbf{r}} = -G\frac{M}{r^3}\mathbf{r} + \mathbf{k}_s \tag{2.18}$$

where

$$\mathbf{k}_s = \ddot{\mathbf{r}}_E + \ddot{\mathbf{r}}_m + \ddot{\mathbf{r}}_e + \ddot{\mathbf{r}}_o + \ddot{\mathbf{r}}_D + \ddot{\mathbf{r}}_{SP} + \ddot{\mathbf{r}}_A \tag{2.19}$$

Perturbing forces on a near-earth satellite are in particular(Seeber, 1993)

1. accelerations due to the non-spherically and inhomogeneous mass distribution within the Earth, $\ddot{\mathbf{r}}_E$
2. accelerations due to other celestial bodies(Sun, Moon and planets), mainly $\ddot{\mathbf{r}}_s$, $\ddot{\mathbf{r}}_m$
3. accelerations due to earth and oceanic tides $\ddot{\mathbf{r}}_e$, $\ddot{\mathbf{r}}_o$
4. accelerations due to the atmospheric drag $\ddot{\mathbf{r}}_D$
5. accelerations due to direct and earth-reflected solar radiation pressure $\ddot{\mathbf{r}}_{SP}$, $\ddot{\mathbf{r}}_A$

Expressions for the perturbing forces can be found in Seeber (1993). An easy to understand description of the main perturbations and its implications can be found in Sellers et. al (2000)

2.5.1 Disturbed Motion due to the Earth's Oblateness

For a near-earth satellite the dominant, by far, Perturbing force is due to the Earth's oblateness. This perturbation is called the *J2 effect*. The effect of the equatorial bulge slightly perturbs the longitude of ascending node Ω and the argument of perigee ω. The perturbations are approximately given by(Sellers, 2000):

$$\dot{\Omega} \approx \frac{-2.06474\cdot 10^{14}}{a^{-7/2}(1-e^2)^2}\cos i \tag{2.20}$$

$$\dot{\omega} \approx \frac{1.03237\cdot 10^{14}}{a^{-7/2}(1-e^2)^2}(4-5\sin^2 i) \tag{2.21}$$

Both $\dot{\Omega}$ and $\dot{\omega}$ are given in [deg/day]. For a polar orbit ($i = 90°$) we note that $\dot{\Omega} = 0$.

2.6 Predicting orbits

An *orbit propagator* is a mathematical algorithm for predicting the future position and velocity (or orbital elements) of an orbit given some initial conditions and assumptions. There are many techniques and methods available, with widely different accuracy and applications.

Most orbit propagators try to solve the extended equation of motion

$$\ddot{\mathbf{r}} = -G\frac{M}{r^3}\mathbf{r} + \mathbf{k}_s. \tag{2.22}$$

The equation can be solved by analytical or numerical orbit integration. In the analytical orbit integration an attempt is made to find algebraic expressions for all acting forces of interest, and then to integrate them in a closed form. This can be done without difficulties for the undisturbed motion. Problems arise when perturbations are considered. For highly accurate orbit prediction a numerical method must to used.

In the numerical orbit integration all forces acting at a satellite position are explicitely calculated, and are then used as starting conditions for a step-wise integration using a numerical integration method like the Runge-Kutta method. The numerical methods are simple and has universal applicability when compared to analytical methods. One disadvantage compared with analytical methods is that in order to predict a future position at time t_{fut} a lot of unwanted positions has to be calculated first. With an analytical method the position can be calculated in one step.

2.6.1 A J2 Propagator

Below an algorithm is given for a simple, two body propagator when only the J2 perturbations are considered(Sellers, 2000). See section 2.5.1. As input the initial classical orbitals elements COEs are given. The index i means initial value, while the index f means future value. All angles are given in radians.

1. Find how the COEs changes due to perturbations. When only the J2 effect is considered, only Ω and ω are affected.

$$\begin{aligned} a_f &= a_i && (2.23) \\ e_f &= e_i && (2.24) \\ i_f &= i_i && (2.25) \\ \Omega_f &= \Omega_i + \dot{\Omega}\Delta t && (2.26) \\ \omega_f &= \omega_i + \dot{\omega}\Delta t && (2.27) \\ & && (2.28) \end{aligned}$$

   $\dot{\Omega}$ and $\dot{\omega}$ are given by (2.20) and (2.21) respectively.

2. Compute initial eccentric anomaly E and initial mean anomaly M. Note that E_i must be in the same quadrant as ν.

$$\begin{aligned} E_i &= \cos^{-1}\left(\frac{e+\cos\nu}{1+e\cos\nu}\right) && (2.29) \\ M_i &= E_i + e\sin E_i && (2.30) \end{aligned}$$

3. Find M_f with

$$M_f = M_i + n\Delta T - 2k\pi, \quad n = \sqrt{\frac{\mu}{a^3}}. \tag{2.31}$$

   In the real world a changes over time due to drag, but this is not considered here.

4. Solve the equation

$$E_f = M_f + e \sin E_f. \tag{2.32}$$

   Note that E_f has to be in same quadrant as M_f For small eccentricities, the following iteration yields a very fast solution

$$\begin{aligned} E_0 &= M_f \\ E_j &= M_f + e \sin E_{j-1}, \quad j = 1, 2, \cdots \end{aligned} \tag{2.33}$$

5. Find ν_f using

$$\nu_f = \cos^{-1}\left(\frac{\cos E_f - e}{1 - e\cos E_f}\right) \tag{2.34}$$

   Again we must ensure that ν_f is in the same quadrant as E_f.

The algorithm above is not especially accurate, but for our purposes it is adequate.

Chapter 3

Orbital considerations

The choice of which orbit the satellite will obtain is mostly determined by the satellite's mission. The ncube's mission is to observe changes at the Earth's surface and therefor a polar orbit is chosen. A polar orbit is when the satellite travels over the north and south poles. To obtain a polar orbit the satellite's inclination, i has to be equal to $90°$ with respect to the equatorial plane, and the eccentricity, e equals zero, see figure 3.

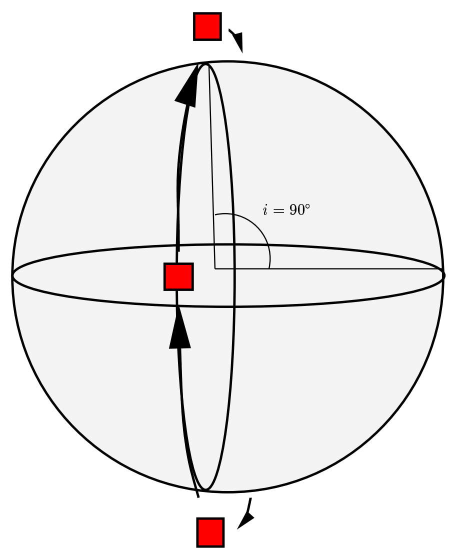


Figure 3.1: Polar orbit

When the eccentricity is chosen to be zero, the orbit will characterize a circle. Satellite's in polar orbits are well suited for scientific purposes. It is preferable to use a polar orbit, since only one satellite can cover the entire world. There are three major classes of polar orbits, dusk-dawn orbit, midafternoon-night orbit and noon-midnight orbit.

Midafternoon-night orbits are mostly used for GPS-navigation, and has semimajor axis, a=20232 Km, period=12 hours, $i \cong 55°$, and $e \cong 0°$ (Sellers, 2000).

In a noon-midnight orbit the satellite will spend approximately one-third of it's period time in the Earth's shadow. This is not preferable for small satellite's that is dependent on as much sunlight as possible to generate enough electrical power. A noon-midnight orbit is a good estimate on the worst case scenario with respect to producing electrical power.

In a dusk-dawn orbit the satellite stays continously within the sunlight and never goes into the Earth's shadow. Dusk-dawn orbits are mainly used for remote-sensing missions.

Ncube's payload has to rely on solar energy to function properly and the area of solar cells is very limited. It is very important that the satellite spends as much time as possible in direct sunlight. The ncube's payload and other function, like data processing and communication needs energy to be operational regardless of the satellite's position. Therefore the satellite must be equipped with batteries that can store electrical power, so that the satellite's equipment can use this stored power whenever it's needed. The selection of orbit has dramatic consequences on the satellite's ability to produce electrical power. In the following sections different orbits and the parameters that determine them will be considered.

3.1 Earth's angular radius

The satellite's altitude, r determines the Earth's angular radius, ρ (Sellers, 2000), see figure 3.2

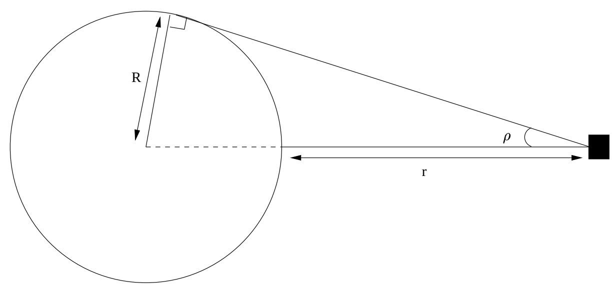


Figure 3.2: Earth's angular radius

For the Earth's angular radius ρ yields:

$$\rho = \sin^{-1}\left(\frac{R}{r+R}\right) \tag{3.1}$$

where; ρ = the Earth's angular radius, (degrees)
R = the Earth's equatorial radius, (6378 Km)
r = the satellite's altitude, (Km)

Figure 3.3 show the Earth's angular radius, calculated for different satellite altitudes. It's seen from figure 3.3 that for a satellite altitude r equal 600 Km, the Earth's angular radius ρ equals 66°

3.2 Maximum time of eclipse

With an expression for the Earth's angular radius and the orbital period, see equation (1.15), it's possible to derive an expression for the total time, TE, a satellite in polar orbit will be in the Earth's shadow. This is very important with respect to the amount of electrical power

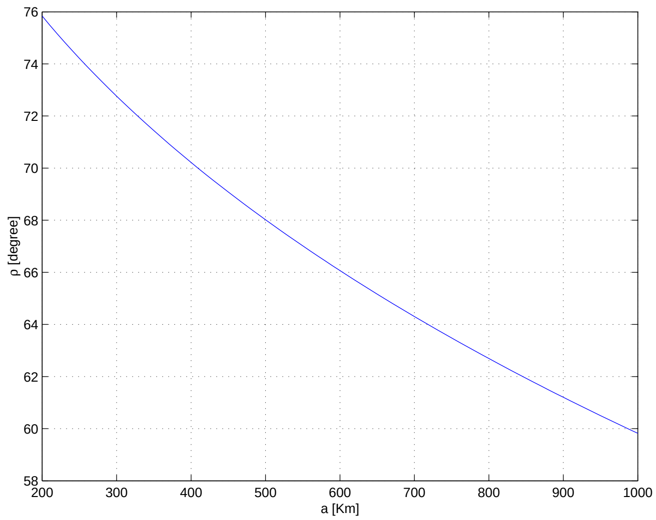


Figure 3.3: ρ for different satellite altitudes

the satellite can produce. The maximum time of eclipse is given by. (Sellers, 2000)

$$TE = P \cdot \frac{2\rho}{360^\circ} \tag{3.2}$$

where;
TE = the maximum time of eclipse, (min)
ρ = the Earth's angular radius, (degrees)
P = the satellite's orbital period, (min)

Figure 3.4 show the total time spent in the Earth's shadow, when a noon-midnight orbit is selected for the satellite.

3.3 Dusk-dawn orbit

In a dusk-dawn orbit the satellite stays continously within the sunlight. The Earth's oblateness will have an effect on the perigee, $\dot{\omega}$ and the nodal regression, $\dot{\Omega}$. Due to this effect it is possible to achieve a dusk-dawn orbit. The ecliptical component of the orbital plane must remain perpendicular to a line connecting the Earth and Sun.

Since the Earth has a eastward nodal progression at inclination greater than 90°, the satellite's orbit must compensate this motion. This can be achieved when $\dot{\Omega}$ compensates the annual motion of the Earth around the Sun, i.e. when;

$$\dot{\Omega} = \frac{360^\circ}{365.24} = 0.98565^\circ \text{per day} \tag{3.3}$$

To compensate for the Earth's nodal motion around the Sun and obtain a dusk-dawn orbit, it can be seen from equation (2.20) that the inclination i, the eccentricity e, and the satellite's

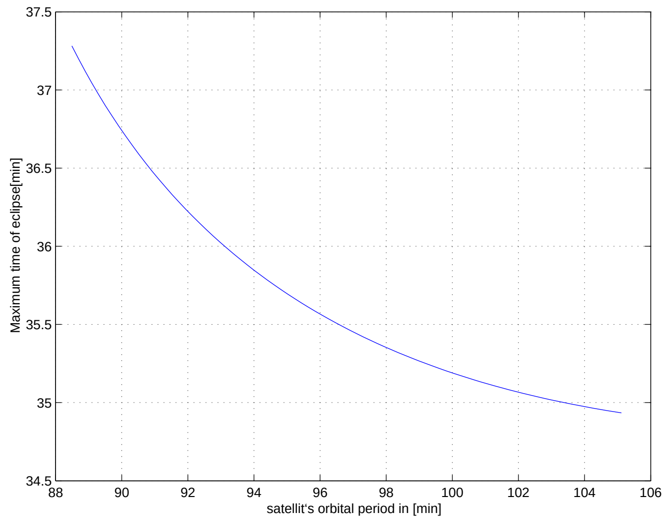


Figure 3.4: Maximum time of eclipse

altitude r, has to be selected properly. Both the eccentricity, and the satellite's altitude, are predetermined, so the only freely selectable is the inclination, see figure 3.5

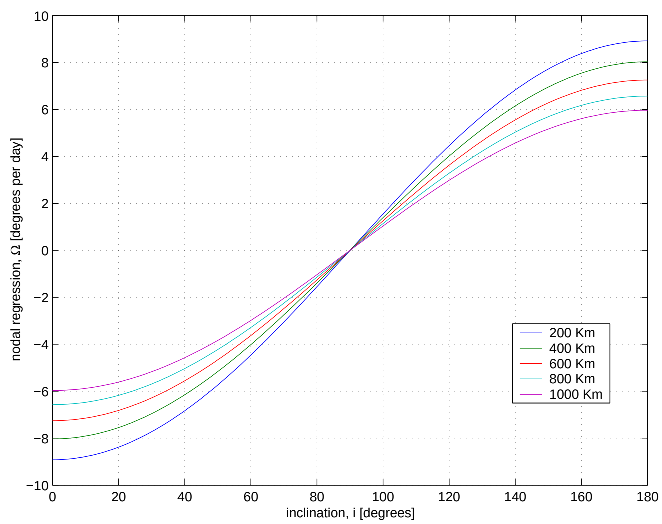


Figure 3.5: Inclination vs nodal regression

From figure 3.5 it can be seen that for a satellite with r equal 600 Km, the inclination i has to be chosen to approximately 98°. This inclination will compensate for the Earth's annual motion around the Sun and the satellite will never go into the Earth's shadow. In figure 3.5 positive numbers for $\dot{\Omega}$ represent eastward movement and negative numbers for $\dot{\Omega}$ represent westward movement. The nodal regression, due to the Earth's equatorial bulge, is zero when the inclination, $i = 90°$. It is also seen that for higher satellite altitude, the effect of the Earth's equatorial bulge get smaller.

In a dusk-dawn orbit the satellite will be continously in the sunlight, and the satellite can produce electrical power all the time. With launch from Kazakhstan the satellite orbit can achieve inclination between 60° and 120°, so the inclination should not be a problem. To compensate for the Earth's nodal motion around the Sun, the satellite need complicated attitude control. It's possible that such attitude control would not be available. It is probably to optimistic to think that the ncube will end up in a dusk-dawn orbit.

3.4 Midafternoon-night orbit

In a midafternoon-night orbit the nodal regression, $\dot{\Omega}$ is the same as for a dusk-dawn orbit. In other words, the $\dot{\Omega}$ will compensate the earth eastward nodal progression, so the satellite's orbital plane is perpendicular to a line connecting the Earth and the Sun. The inclination is less than 90° so a midafternoon-night orbit is obtained. Due to inclination less than 90°, the satellite will periodicly go into the earth's shadow.

If the inclination is chosen to near 90° or greater, the satellite will obtain a dusk-dawn orbit. As the inclination decreases the satellite will spend more and more time in the Earth's shadow. Now the satellite has to rely on stored power from batteries when it goes into the shadow regions. The midafternoon-night orbit is also dependent on a good attitude control to compensate for the Earth's nodal motion. So this orbit is probably to optimistic as well.

3.5 Noon-midnight orbit

In a noon-midnight orbit the satellite will follow a day-night cycle almost like our day and night cycle. This orbit doesn't compensate for the Earth's nodal motion around the Sun, the $\dot{\Omega}$ will follow the earth's movement. In other words the $\dot{\Omega}$ will change with a few degrees per 24 hours. The inclination is $\pm 90°$. This means that the satellite will spend about one-third of it's orbital time in the Earth's shadow and the satellite has to depend on stored electrical power when it's not able to produce power. The noon-midnight orbit is not dependent on such attitude control as the two previous orbits. It is feasible to belive that the satellite will go in some combination of dusk-dawn and noon-midnight orbit if the inclination is chosen to $\pm 90°$. For the smallest amount of time spendt in sunlight, the noon-midnight orbit is a good estimate of worst case scenario.

Chapter 4

Power Production

4.1 Introduction

The Sun is the energy source of our satellite. We can convert the solar energy to electrical energy by using the incoming solar photons to create a flow of electrons. This way of conversion is possible by the use of solar cells, or *photovoltaic*, PV, cells. When sunlight shines on the solar cell, electrical current flows.

The energy produced by the satellite is determine mainly by the area, A, of solar cells illuminated by sunlight, the intensity of the sunlight, I_0, the incident angle, ξ, and the energy-conversion efficiency of the solar cells, $\eta(\xi)$.

$$E = A_{eff} \cdot I_0 \cdot \eta(\xi) \tag{4.1}$$

where I_0 is the incoming light intensity, given by $1358\mathrm{W/m^2}$ near the Earth (Sellers, 2000).

4.2 Area illuminated

We use the *angle of incidence*, ξ, to determine the area of solar cells that is illuminated by the sun light. This area is called the effective area. This angle lies between a line perpendicular to the surface and the Sun vector.

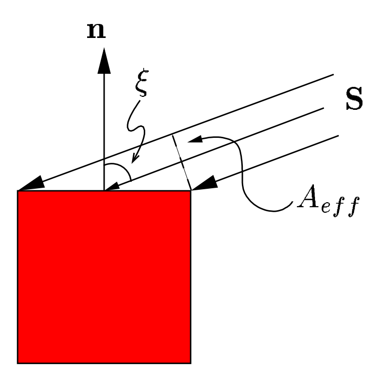


Figure 4.1: Incident angle and effective area.

As we can see from figure 4.1, the effective area can be expressed as

$$A_{eff} = A_0 \cdot \cos\xi \tag{4.2}$$

where

A_0	=	The area of solar cells on one side of the satellite
ξ	=	The incident angle

4.3 Energy-conversion efficiency

The *energy-conversion efficiency* of a solar cell, η, measures how efficiently the solar cell converts solar energy into useful electrical energy.

In the documentation from manufacturers of solar cells, a value for energy-conversion efficiency is mentioned. ncube will probably be equipped with solar cells where this efficiency is approximately 26 percent.

The documented energy-conversion efficiency is the efficiency that is valid for 0 degrees incidence angle. That is, when the light hits perpendicular on the surface. However, the efficiency varies with the incidence angle, ξ.

The "effective" energy conversion efficiency is given by:

$$\eta(\xi) = \eta_0 \frac{I_{trans}(\xi)}{I_{trans}(0)} = \eta_0 \frac{T(\xi)}{T_0} \tag{4.3}$$

where

η_0	=	The energy-conversion efficiency at normal incidence($\xi = 0$)
$T(\xi)$	=	The transmission coefficient at incident angle ξ
T_0	=	The transmission coefficient at normal incident angle

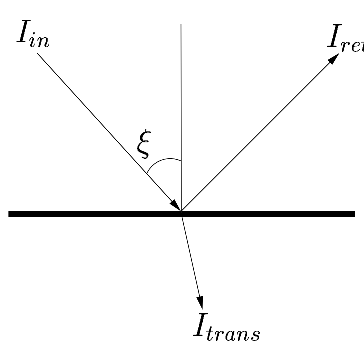


Figure 4.2: Light crossing a surface.

The transmission coefficient

When light hits a surface the following boundary conditions is obeyed:

$$\begin{aligned}
\epsilon_1(E_{01}+E_{0R})_z &= \epsilon_2(E_{0T})_z \\
(B_{0I}+B_{0R})_z &= (B_{0T})_z \\
(E_{0I}+E_{0R})_{x,y} &= (E_{0T})_{x,y} \\
\frac{1}{\mu_1}(B_{0I}+B_{0R})_{x,y} &= \frac{1}{\mu_2}(B_{0T})_{x,y}
\end{aligned}$$

where B_0 and E_0 are magnetic and electric amplitudes, the subscripts I, R and T relates to the incoming, reflected and transmitted rays and ϵ and μ are the permittivity and permeability of the two materials (Griffiths,1981).

The transmission coefficient is given by the above mentioned boundary conditions. The result depends on the polarization of the light.

Parallel polarization

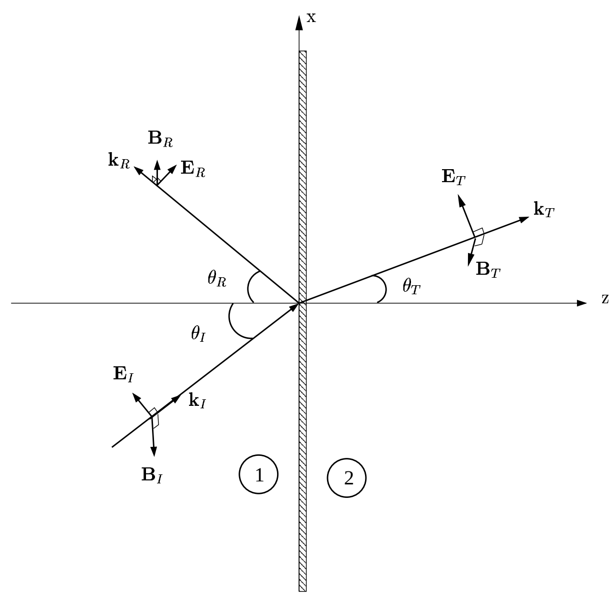


Figure 4.3: Parallel polarization.

When the polarization is parallel to the plane of incidence, the boundary conditions become:

$$\begin{aligned}
\epsilon_1(-E_{0I}\sin\theta_I + E_{0R}\sin\theta_R) &= \epsilon_2(-E_{0T}\sin\theta_T) \\
0 &= 0 \\
E_{0I}\cos\theta_I + E_{0R}\cos\theta_R &= E_{0T}\cos\theta_T \\
\frac{1}{\mu_1}(B_{0I}-B_{0R}) &= \frac{1}{\mu_2}B_{0T}
\end{aligned}$$

By solving these equations, and using the laws of reflection and refraction, one finds the transmission coefficient:

$$Tp(\xi) = a(\xi) \cdot b \cdot \left(\frac{2}{a(\xi)+b}\right)^2 \tag{4.4}$$

Where a and b are defined by:

$$\begin{aligned} a(\xi) &= \frac{\sqrt{1-(\frac{n_1}{n_2}\sin\xi)^2}}{\cos\xi} \\ b &= \frac{\mu_1 n_2}{\mu_2 n_1} \end{aligned}$$

Perpendicular polarization

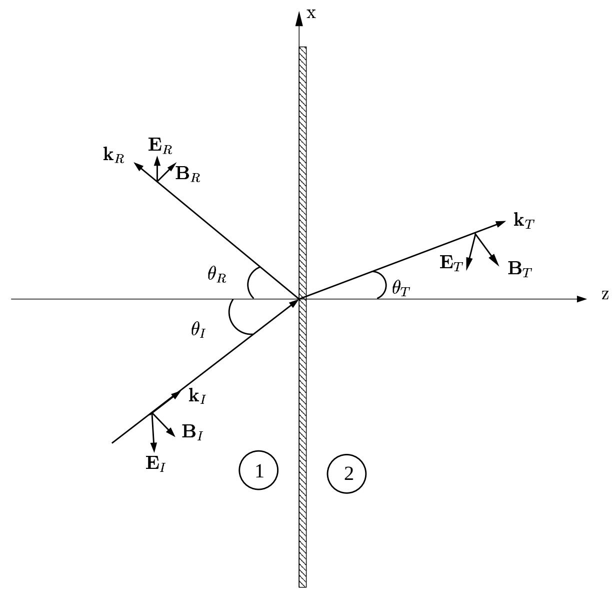


Figure 4.4: Perpendicular polarization.

When the polarization of the light is perpendicular to the plane of incidence, the boundary conditions become:

$$\begin{aligned} 0 &= 0 \\ B_{0I}\sin\theta_I + B_{0R}\sin\theta_R &= B_{0T}\sin\theta_T \\ E_{0I} + E_{0R} &= E_{0T} \\ \frac{1}{\mu_1}(-B_{0I}\cos\theta_I + B_{0R}\cos\theta_R) &= \frac{1}{\mu_2}(-B_{0T}\cos\theta_T) \end{aligned}$$

In the same way as for paralell polarization one can use these equations to find the transmission coefficient.

$$Tn(\xi) = a(\xi) \cdot b \cdot \left(\frac{2}{a(\xi) \cdot b + 1} \right)^2 \tag{4.5}$$

where a and b are defined in the previous section.

Arbitrary polarization

We assume that the sunlight hitting the satellite has some arbitrary polarization. Based on this, we approximate the "total" transmission coefficient as the average of the parallel and perpendicular polarized transmission coefficients:

$$T(\xi) = \frac{Tp(\xi) + Tn(\xi)}{2} \tag{4.6}$$

With "typical" values for two materials (for instance glass and air), the energy-conversion efficiency will vary with the incidence angle like in figure 4.5. As one can see from the graph, the efficiency is relatively constant for small angles, but decreases rapidly when the incidence angle becomes more than approximately 80 degrees.

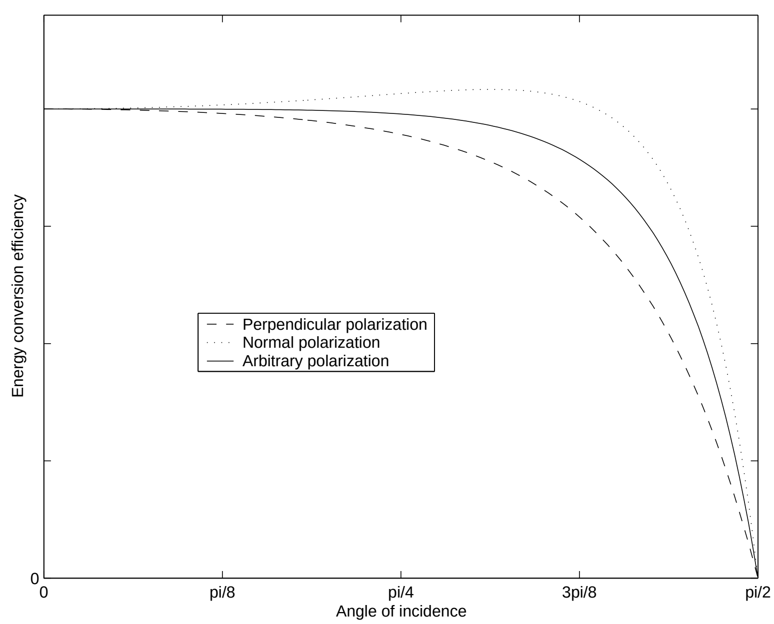


Figure 4.5: Energy-conversion efficiency.

4.4 Angles of incidence

To calculate the power output from the solar cells, we need some definitions. We want to consider the energy produced by the different panels of the satellite. To do so, we need expressions for the angles at which the sun hit the different panels.
We consider the Sun-Earth-satellite system.

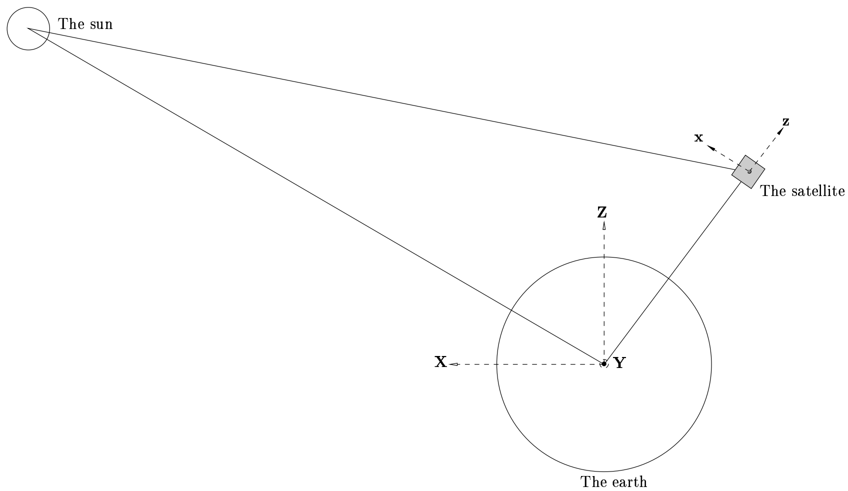
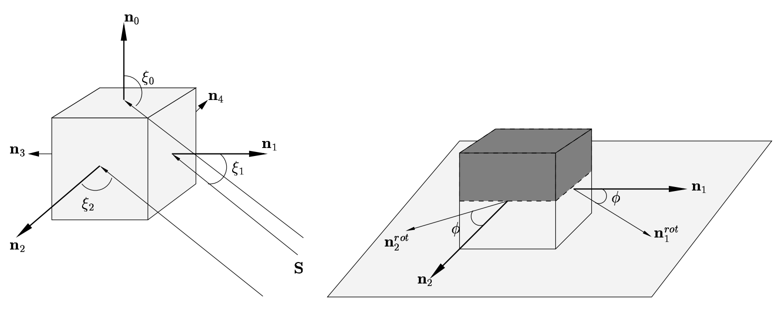


Figure 4.6: The Sun-Earth-satellite system.

As the satellite rotates about itself, the incident angles will vary. Figure 4.7 indicates the angles of incidence and the rotation angle.



(a) Angles og incidence. (b) Rotation angle.

Figure 4.7: Angles of incidence and rotation.

We define the direction of the incoming light, and the axis directions of the 5 panels of the satellite.

$$\mathbf{S} = [a \;\; b \;\; c] \; ; \; \sqrt{a^2+b^2+c^2} = 1$$

$$\begin{aligned}
\mathbf{n}_0 &= [0 \;\; 0 \;\; 1]^T \\
\mathbf{n}_1 &= [\cos\phi \quad \sin\phi \;\; 0]^T \\
\mathbf{n}_2 &= [\sin\phi \quad \cos\phi \;\; 0]^T \\
\mathbf{n}_3 &= [-\cos\phi \quad -\sin\phi \;\; 0]^T \\
\mathbf{n}_4 &= [-\sin\phi \quad -\cos\phi \;\; 0]^T
\end{aligned}$$

The angles in the figure above can now be found, by using the property of the inner product.

$$\mathbf{S}\cdot\mathbf{n}_0 = |\mathbf{S}|\cdot|\mathbf{n}_0|\cos\xi_0 \implies \xi_0 = \cos^{-1}(\mathbf{S}\cdot\mathbf{n}_0) \tag{4.7}$$
$$\mathbf{S}\cdot\mathbf{n}_1 = |\mathbf{S}|\cdot|\mathbf{n}_1|\cos\xi_1 \implies \xi_1 = \cos^{-1}(\mathbf{S}\cdot\mathbf{n}_1) \tag{4.8}$$
$$\mathbf{S}\cdot\mathbf{n}_2 = |\mathbf{S}|\cdot|\mathbf{n}_2|\cos\xi_2 \implies \xi_2 = \cos^{-1}(\mathbf{S}\cdot\mathbf{n}_2) \tag{4.9}$$
$$\mathbf{S}\cdot\mathbf{n}_3 = |\mathbf{S}|\cdot|\mathbf{n}_3|\cos\xi_3 \implies \xi_3 = \cos^{-1}(\mathbf{S}\cdot\mathbf{n}_3) \tag{4.10}$$
$$\mathbf{S}\cdot\mathbf{n}_4 = |\mathbf{S}|\cdot|\mathbf{n}_4|\cos\xi_4 \implies \xi_4 = \cos^{-1}(\mathbf{S}\cdot\mathbf{n}_4) \tag{4.11}$$

4.5 Energy production

The energy produced is found by multiplying the effective intensity that hits the panels with the effective area. The area of the solar panels will lie between 0.0064m^2 and 0.0080m^2, depending on the construction. The energy becomes

$$E = I_0 \cdot \eta(\xi) \cdot A_0 \cdot \cos(\xi) \tag{4.12}$$

The total energy is found by adding up the contributions of all the panels that are in the sun:

$$E_{tot} = \sum_{i=0}^{4} E_i$$

The general expression for the energy produced becomes

$$E_{tot} = (A_0 \cdot I_0) \cdot \sum_{i=0}^{4} \eta_i \cdot \cos\xi_i$$

where the energy conversion coefficient is given by

$$\eta_i = \begin{cases} \eta(\xi_i) & \text{if } \xi_i \le \frac{\pi}{2}, \\ 0 & \text{else.} \end{cases}$$

In our simulator we calculate the energy produced during a certain time interval, Δt. We approximate by assuming that the satellite stays in the same place during this time interval, spinning around itself. Then, for the next interval the satellite is in another place, spinning. We find the angular energy produced, and then we integrate with respect to the rotation angle ϕ.

To find the upper limit of our integral we need an expression for how many radians the satellite spins during the time interval. This is given by

$$2\pi \cdot f_s \cdot \Delta \mathrm{t}$$

where f_s denotes the satellites spin frequency.
The final expression for the energy produced by our satellite during the time interval Δt, is

$$E_{tot} = \left(\frac{A_0 \cdot I_0}{2\pi \cdot f_s}\right) \int_0^{(2\pi \cdot f_s \cdot \Delta \mathrm{t})} \left(\sum_{i=0}^{4} \eta(\xi_i) \cos \xi_i\right) d\phi \tag{4.13}$$

4.6 Complicating factors

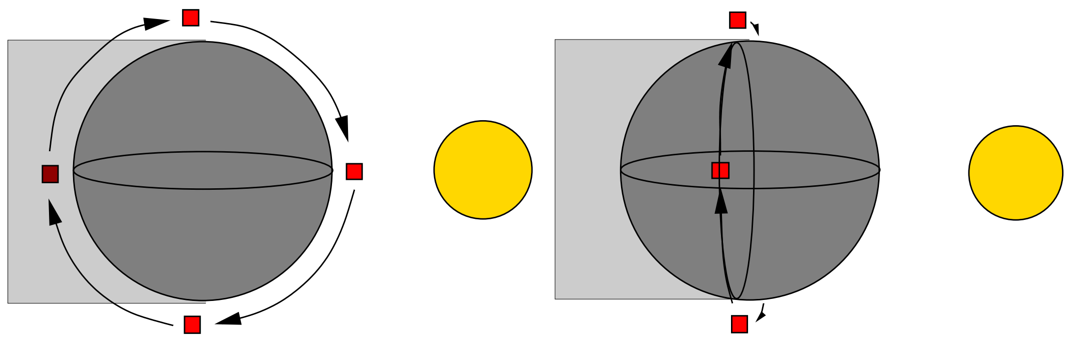
There are several environmental factors that can degrade solar cell performance, but these will not be considered. We will however mention some of them.

i)Temperature.
The solar cells are most efficient at low temperatures and loose efficiency at higher temperatures. Typical solar cells lose from 0.025 to 0.075 percent of their efficiency per $^\circ C$ as the temperature increases above $28^\circ C$(J.J.Sellers). This means that thermal control is very important.

ii)Radiation and charged particles.
As radiation and particles hit the soalr arrays, the materials begin to degrade. Depending on the orbit, solar arrays can lose up to 30 percent of their effectiveness over ten years(J.J.Sellers). We do not expect our satellite to live long enough for this to have any influence.

4.7 Preliminary results

There will be many instruments that require energy in our satellite, so we want an estimate of the energy available in approximately best and worst case scenarios. Both cases occur at an inclination angle of 90°. Both cases are shown in figure 4.8.



(a) Noon-midnight orbit. (b) Morning-evening orbit.

Figure 4.8: "Worst" and "best" case scenarios.

4.7.1 Best case scenario.

For the best case scenario the satellite is said to follow a morning-evening orbit. The satellite will be in the sun all the time, and the four side panels of the satellite are the only panels that are exposed to the sun. We make some assumptions to simplify:

$$\begin{aligned} \eta(\xi_0) &= \eta(\xi_1) = \eta(\xi_2) = 0.26 = \eta \\ A_0 &= 0.0080\mathrm{m}^2 \\ \Delta\mathrm{t} &= 1\mathrm{s} \\ \mathrm{f}_s &= 1\mathrm{Hz} \end{aligned}$$

That is we assume the conversion coefficient to be constant which will overestimate the energy produced. We also assume the area of the solar panels to be the maximum area. So the energy our satellite will produce per second in the best case becomes

$$\begin{aligned} E_{best} &= 4 \cdot \left(\frac{A_0 \cdot I_0}{2\pi}\right) \cdot \eta \cdot \int_0^{\pi/2} [\cos\xi_1 + \cos\xi_2] d\phi \\ &= 3.596W \end{aligned}$$

So our satellite will produce 3.596W, day and night. The satellite might actually produce more than this for some other orbits, at certain times, but will also produce much less at others, 0W if in shadow for some part of an orbit.

4.7.2 Noon-midnight: worst case

For the worst case scenario the satellite is said to follow a noon-midnight orbit. The satellite will be in the sun for approximately 930 minutes a day, and all sides are exposed. We make some assumptions here as well.

$$\begin{aligned}
\eta(\xi_0) &= \eta(\xi_1) \;=\; \eta(\xi_2) \;=\; 0.26 \;=\; \eta \\
A_0 &= 0.0064\mathrm{m}^2 \\
\Delta\mathrm{t} &= 1\mathrm{s} \\
\mathrm{f}_s &= 1\mathrm{Hz}
\end{aligned}$$

In this case the energy that the satellite produces will, ofcourse, vary during the day. We will look at these cases in more detail later, but for now we calculate the minimum and maximum energy produced while the satellite is in the sun. In the maximum case the satellite has three panels in the sun, and the incidence angle with the top panel is $45° \Rightarrow \mathbf{S} = [\frac{\sqrt{2}}{2} \;\; \frac{\sqrt{2}}{2} \;\; \frac{\sqrt{2}}{2}]$. For the worst case $\mathbf{S} = [0 \;\; 0 \;\; 1]$, and our estimates become

$$\begin{aligned}
E_{max} &= 4 \cdot \left(\frac{A_0 \cdot I_0}{2\pi}\right) \cdot \eta \cdot \int_0^{\pi/2} [\cos\xi_0 \cos\xi_1 + \cos\xi_2] d\phi \\
&= 3.632\mathrm{W} \\
E_{min} &= 4 \cdot \left(\frac{A \cdot I_{in}}{2\pi}\right) \cdot \eta \cdot \int_0^{\pi/2} [\cos\xi_0] d\phi \\
&= 1.439W
\end{aligned}$$

So the satellite will at best produce 3.632W and at worst 1.439W, while in the sun, in the worst case orbit.
These numbers are approximations. We have made som assumptions to calculate these. We will show later, through our simulator these results without these simplifications. But these numbers are good indicators of the energy that will be produced.

Chapter 5

The CubeSim Simulator

In the preceding chapters the theory necessary to predict the satellite's position and available energy has been presented. This chapter briefly explains how the theory was implemented in a simulation program, *CubeSim*, using MATLAB. Source code and documentation can be found in appendix A and B.

5.1 Outline of the Simulator

The simulation process is divided into different steps for better modularity. As input the initial classical orbital elements and solar panel parameters are given.

1. The satellite is propagated and its position is determined at each time step. The two-body propagator described in section 2.6.1 is used. The propagator gives the satellite's positions in COEs so equation (2.16) is used to convert the positions to the ECI frame.

2. The positions calculated in the previous step are analyzed and checked if the satellite is in the Earth's shadow or not. The method used is the simple cylindric shadow model described in section 2.2.3.

3. The unit vectors to the Sun are transformed from the ECI frame to the orbit frame using the transformations given in equation (2.1).

4. The most time-consuming step is to calculate the generated energy at each time step. This is done by solving the integral given by (4.13). The integral is difficult to solve analytically due to the non linear expression for the energy-conversion efficiency $\eta(\xi)$. A numerical integration method is therefore used.

5. In the last step the data generated in the previous steps are analyzed and plotted. Total energy production and mean power generation is calculated using a Riemann sum.

5.2 Assumptions and Simplifications

When designing the simulator a number of assumptions and simplifications had to be done. Most of them are already mentioned and discussed in the previous chapters but restated here for completeness.

- The z-axis of the body frame is the same as in the orbit frame, $z_b = z_o$. Therefore the bottom side of the satellite will always face the Earth.
- The side facing the Earth has no solar panels. The other five sides have solar panels with the same area A_0.
- The satellite has a constant spin about the z_b-axis. In the simulator this spin is set to 2π rad/s (One revolution per second).
- When calculating the generated power at each time step, the sunlight's angle of incidence is assumed to be constant over a period of 1 second. In reality the angle changes approximately $\left(\frac{360}{97 \cdot 60}\right) = 0.06°$, which is negligible.
- When seen from the Sun, the satellite can be assumed to be placed in the center of the Earth.

Chapter 6

Simulations and Results

The power generation was simulated for different scenarios. All simulations were done with a time step of $\Delta t = 30$ seconds and simulated in 97 minutes(one revolution). Parameters common to all simulations are shown in table 6.2. During all simulations start time was set to 0. The different scenarios are summarized in table 6.1. A discussion of the simulations is given at the end of the chapter.

6.1 Scenarios

Scenario I An morning-evening orbit. The satellite stays in sunlight during the whole orbit.

Scenario II An day-midnight orbit.

Scenario III Also an morning-evening orbit, but with inclination set to $i = 80°$.

Scenario IV An morning-evening orbit as in scenario I, but with the Sun having a tilt of $\epsilon_s = 23°$.

Scenario V An day-midnight orbit with $\Omega = 60°$.

Table 6.1: Simulation scenarios

Scenario	i	Ω	ϵ_s
I	90°	90°	0°
II	90°	0°	0°
III	80°	90°	0°
IV	90°	90°	23°
V	90°	60°	0°

Table 6.2: Simulation parameters

Parameter	Value
Semimajor axis, a	$6.9713 \cdot 10^3$ m
Eccentricity, e	0.0001
Argument of perigee, ω	90°
True anomaly, ν	0°
Energy-conversion efficiency, η_0	0.26
Power intensity, I_{in}	1358 W/m^2
Solar cell area, A_0	0.0060, 0.0080 m^2
Index of refraction, n_2	3.5
Time step, Δt	15 s
Steps	388

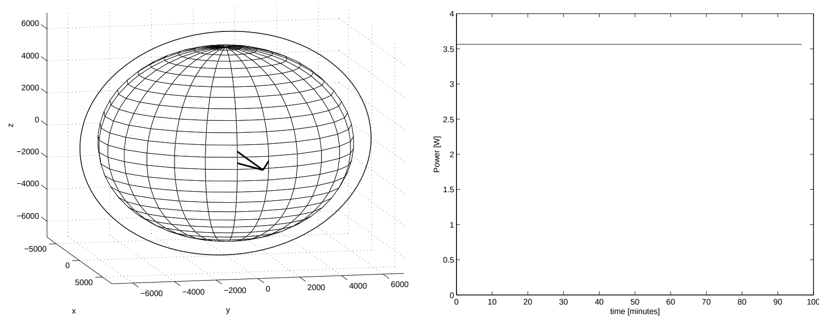
6.2 Results

The results of the simulations are shown in figure 6.1-6.5 and table 6.3. Shadow is indicated with a thick line. The columns in table 6.3 are:

- T_s Percent of the time spent in shadow.
- P_m Mean power production during one orbit. Calculated for an solar cell area of $A_0 = 0.0060$ m^2 and $A_0 = 0.0080$ m^2 respectively.
- E_t Total energy production during one orbit. Also calculated for two different solar cell areas.

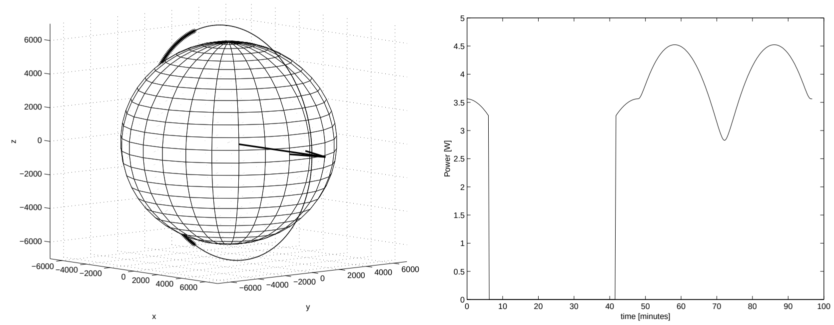
Table 6.3: Simulation results

Scenario	T_s [%]	P_m[W]	E_t [kJ]
I	0.0	2.67 - 3.56	15.55 - 20.74
II	36.6	1.84 - 2.46	10.75 - 14.34
III	0.0	2.73 - 3.65	15.94 - 21.26
IV	0.0	2.80 - 3.74	16.32 - 21.79
V	21.7	2.30 - 3.07	13.43 - 17.91



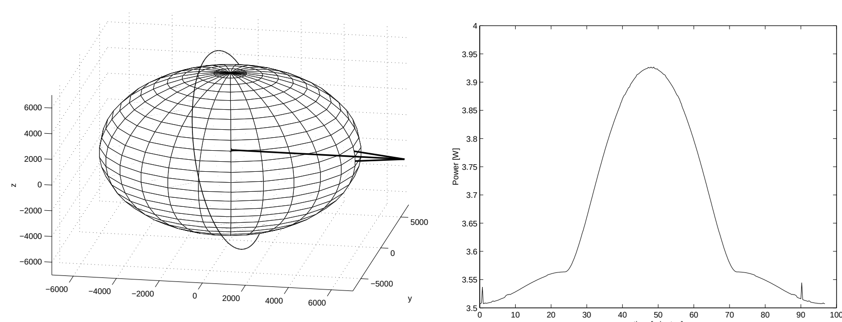
(a) (b)

Figure 6.1: Simulation of scenario I



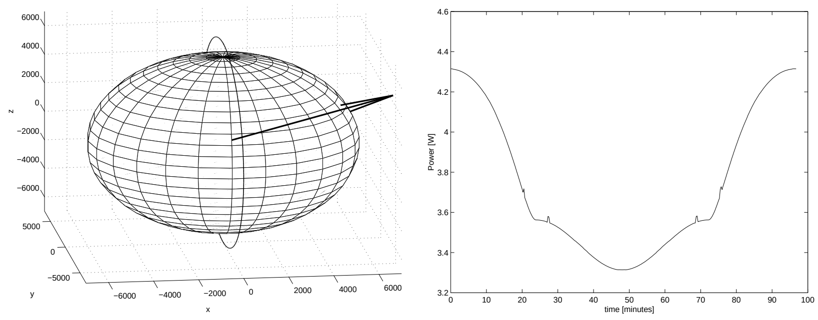
(a) (b)

Figure 6.2: Simulation of scenario II



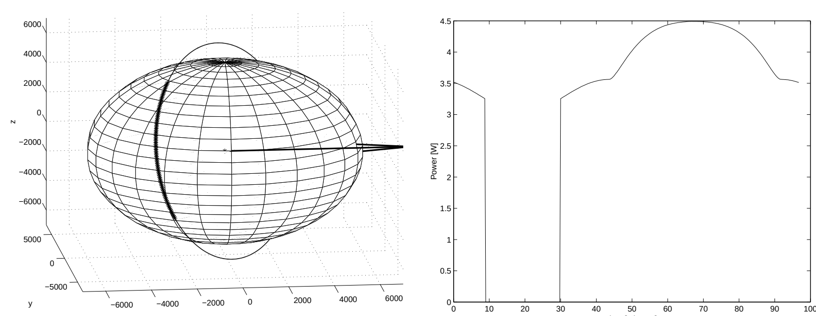
(a) (b)

Figure 6.3: Simulation of scenario III



(a) (b)

Figure 6.4: Simulation of scenario IV



(a) (b)

Figure 6.5: Simulation of scenario V

6.3 Discussion

The simulations show some interesting results. As expected the power generation is constant in scenario I. In scenario I the top panel of the satellite doesn't produce any energy due to an angle of incidence $\xi_0 = 90°$.

A more interesting result was obtained in scenario II, which is a day-midnight orbit. Approximately 36% of one revolution is spent in shadow, therefore giving a low mean power production. Another interesting aspect is the shape of the power plot. The plot has a sinusoidal shape with a peak production of approximately $P_m = 4.5$ W. This peak happens when all solar panels are visible and has an angle of incidence of $\xi = 45°$. A minimum production, $P_m = 2.8$ W, is obtained when only the top panel is visible to the sun.

Scenario III is a special case of a morning-evening orbit. The simulations showed a slightly larger energy production than in scenario I. The reason is that the top panel is visible in parts of the orbit, though the angle of incidence is quite high.

Scenario IV was simulated to investigate the effect of the sun's elevation angle on a dusk-dawn orbit. As shown in section 2.2.2 the elevation angle changes $\pm 23°$ over a year. This scenario gave the highest energy production of all simulated scenarios. The variations of the Sun's elevation angle will probably don't have a negative effect on the energy production. The spikes shown in figure 6.4.b are probably due to singularities in the numerical integration method.

Over a year the Earth will revolve once around the Sun. If the satellite doesn't have a sun-synchronous orbit, the orbit will inevitable become a day-night orbit. Scenario II is the worst case, while scenario V spends 21% of one revolution in shadow.

All the simulated scenarios are feasible for the ncube satellite. During one year the satellite will probably experience all of them. Scenario II showed that the optimal energy production is achieved when one edge of the cube is pointed towards the sun. By using a sun sensor and a good attitude control system the energy production can be increased. How much this will cost in additional power consumption we don't know. We also note that the simulations agrees with the prelimenary results in section 4.7.

Chapter 7

Final Conclusions and Recommendations

The main purpose of our project was to make a tool to estimate the energy available to the satellite. Many parameters and properties of the satellite are still unknown at this stage, but from our simulations we can conclude that *a mean power production of 2.4-3.5 W is feasible.* This numbers must be considered as an rough estimate, but give an important insight into what is possible to achieve with the satellite. The simulations also revealed that a more optimal attitude control system can improve the amount of available energy.

7.1 Recommendations for Future Work

During completion of this report we have a feeling that we have only seen "the top of the iceberg" of the topics covered. Additional work should be done in most areas.

- A better satellite propagator should be implemented, accounting for the different perturbing forces. A good choice will be the widely used MSGP-4[1] propagator.

  The propagator implemented in our simulator is a good tool for the investigation of special cases, but fails when long term effects are considered.

- The satellite's attitude model should be extended and improved. A model formulation using quaternions will give a non-singular representation and full freedom in choice of roll, pitch and yaw angles. This will probably simplify the simulation of different attitudes.

- Optimization of the energy calculation code is necessary, especially if more complex solar panel shapes are considered.

- Use of a more accurate model of the solar panels, accounting for long term effects, temperature and the conversion from solar energy to current and voltage.

- Models for energy consumption should be implemented. We have focused on how much energy there is available, not how it is stored or used.

[1] Merged Simplified General Perturbations-4. MSGP4 is the current generation of operational software used by the U.S. Space Command

- The simulator has an educational value. A simplified version could be made as an applet and presented on the world wide web. This way pupils and students can experiment with different orbits and learn about solar energy.
- An easier to understand graphical users interface and a better presentation of the generated data.

Chapter 8

References

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Appendix A

CubeSim User's Guide

This chapter briefly gives a description of how to use the CubeSim simulator. A screenshot of the GUI is shown in figure A.1.

To get started, start Matlab and change your working path to the directory where the CubeSim files are started. Then type: `simulatorgui` at the command line.

A.1 GUI description

A.1.1 Orbit parameters

The different orbit parameters are described in section 2.4.For numerical reasons the eccentricity should not be set to $e = 0$. A small number can be used instead.

A.1.2 Solar-cell parameters

The solar cell parameters are described in section XX. Note that the index of refraction have to be $n_2 > 1$.

A.1.3 Simulation parameters

Time step Determines how often the satellite's position is updated. The time step must be an integer and $\Delta t \geq 1$. Time step is given in seconds.

Start time Days passed since the first day of spring. This parameter determines where the Sun is when the simulation is started. See section 2.2.2.

 # steps Determines how many time steps, N, are simulated. The total simulation time is given by $T = \Delta t \cdot N$.

Calculate energy The energy calculations can be quite lengthy. Therefore they can be turned off.

Show sun vector If checked, a vector indicating the direction to the sun is drawn in the resulting plot. Only the direction at the start time is drawn.

J2 perturbation Propagates the satellite considering J2 perturbations. Will only have a visible effect in long simulations.

A.2 Output

After running a simulation two plots are generated. One showing the orbit, while the other displays the generated power. Mean power production, total energy production and time spent in shadow are written to the command window.

A.3 Comments

The energy calculations are time consuming. Increase the step size for a faster simulation. Time steps greater than one second will give an error when the total energy is calculated. Experiments has shown that time steps less than 30 seconds give an acceptable result.

Note that the orbit plot can be rotated and viewed from different angles. The source code is available to be modified.

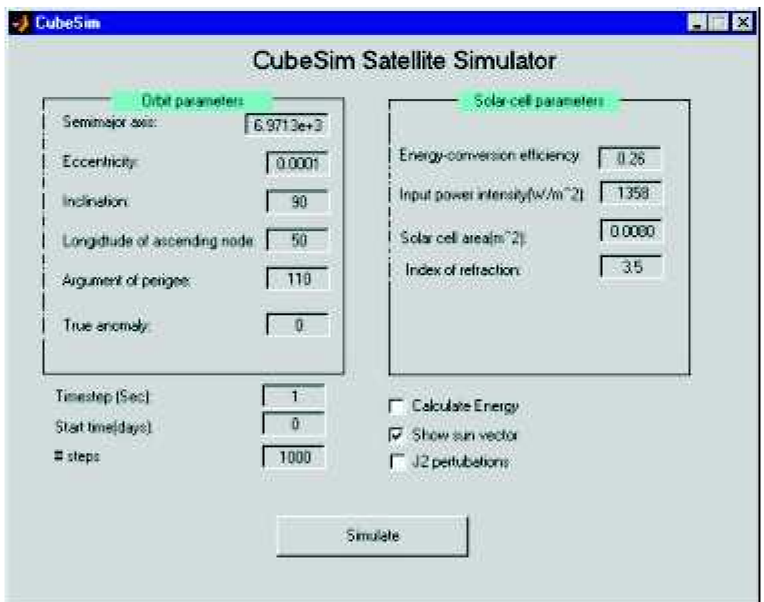

Figure A.1: CubeSim graphical user interface

Appendix B

MATLAB Sourcecode

In this section a selection of the most important source code is presented. For more details refer to the enclosed CD.

Main simulation front-end

```
function DoSimulations(COE_in, start_time, timestep, steps, perturbed,calculate_energy, show_sun)
%Main simulation loop.
%Author: Kjell Magne Fauske
[pos time] = SimulateSatellite(COE_in, start_time, timestep, steps, perturbed);
[sunpos shadowpos shadowcount] = FindInSun(pos, time);
disp('Time spent in shadow:');
timeinshadow=shadowcount*timestep
if timeinshadow >= 0
    disp('Percent spent in shadow:');
    percentshadow=timeinshadow/(steps*timestep)
end
if (calculate_energy==1)
    warning off;
    eff=CalculateEnergy(sunpos, time);
    warning on;
    disp('Energy prduction');
    totalenergy=sum(eff)*timestep
    disp('Mean power');
    meanpower=totalenergy/(steps*timestep)
end

%plotting ------
%Create a spehere as the Earth
figure(1);
[sx sy sz] = sphere(25); %50
r=6000;%6371.32;
mesh(r*sx, r*sy, r*sz);
hold on;

%plot a vector indicating the direction to the sun
if show_sun==1
    T=start_time/(3600*24);
    sunvec=(findSunDirection(T)*9000);
    u=sunvec(1); v=sunvec(2); w=sunvec(3);
    h=quiver3(0,0,0,u,v,w, 'g');
    set(h,'LineWidth',2);
```

```
end
%plot the satellite orbit
h=plot3(pos(1,:),pos(2,:),pos(3,:),'-');
set(h,'LineWidth',1);
%plot the satellites shadow positions
plot3(shadowpos(1,:),shadowpos(2,:),shadowpos(3,:),'r*')

%set appropriate axes
minx= -7000; maxx=7000;
miny= -7000; maxy=7000;
minz= -7000; maxz=7000;
axis([minx maxx miny maxy minz maxz]);
xlabel('x'); ylabel('y'); zlabel('z');

%Plot the power
if calculate_energy==1
    figure(2)
    timescale=(0:timestep:(timestep*steps-1))/60;
    plot(timescale,eff);
    xlabel('time [minutes]');
    ylabel('Power [W]');
    hold on;
end
```

SimulateSatellite.m

```
function [XYZpos ,time] = SimulateSatellite(COE_in, start_time, timestep, steps, perturbed)
%Simulates the satellite motion
%Input:
%   COE_in       - Initial COE position
%   start_time   - Start time of simulation. Given in seconds since first day of spring.
%   timestep     - Simulation timestep in seconds
%   steps        - Number of steps to simulate
%   pertubed = 0 - No pertubations
%            = 1 - Simultion with pertubations.
%Output:
%   COEs    - Satellite positions given in COE
%   XYZpos  - Satellite position given in ECI frame
%   time    - Time vector in seconds
%Author(s): Kjell Magne Fauske

N=steps;
dt=timestep;
T = start_time;       %t_fut - t_in
%allocate memory for variables
pos=zeros(3,N);
time=T:dt:(T+dt*N-1);
COE_i = COE_in;
disp('Calculating satellite positions...');
if perturbed == 0
    for i=1:N,
        COE_fut=propagateSat(COE_i, dt);
        T=T+dt;
        COE_i=COE_fut;
        ecipos=COEtoXYZ(COE_fut);
        pos(:,i)=ecipos;
    end
```

```
else
    for i=1:N,
        COE_fut=propagateSatJ2(COE_i, dt);
        T=T+dt;
        COE_i=COE_fut;
        ecipos=COEtoXYZ(COE_fut);
        pos(:,i)=ecipos;
    end
end
disp('...done.');
%return values
XYZpos=pos;
```

propagatesat.m

```
function COE_fut =propagateSat(COE_in, timestep)
%Propagates the satellite without any pertubations
%
%Input: COE_in   - Initial COE values
%       timestep
%Output COE_fut - future COE values
%Author(s): Kjell Magne Fauske

rad=pi/180;     %convert deg to rad
deg = 180/pi;   %convert rad to deg
mu=3.986*10^5;  %gravitational parameter

a=COE_in(1); e=COE_in(2);
i=COE_in(3); omega=COE_in(4);
Omega=COE_in(5); nu=COE_in(6);

T=timestep;

n=sqrt(mu/(a^3));

%Compute inital eccentric anomaly E
E_in=acos((e+cos(nu))/(1+e*cos(nu)));
E_in=fixQuadrant(E_in, nu);
%Find initial mean anomaly M
M_in=E_in-e*sin(E_in);
M_fut=n*T+M_in;
M_fut=mod(M_fut,2*pi);
E_fut=compE_fut(M_fut,e);
E_fut=fixQuadrant(E_fut, M_fut);
nu_fut=acos((cos(E_fut)-e)/(1-e*cos(E_fut)));
nu_fut=fixQuadrant(nu_fut, M_fut);
COE_fut=[a, e, i, omega, Omega, nu_fut]';
```

CalculateEnergy.m

```
function power = CalculateEnergy(in_sun_pos, time)
%Calculates the total power generated by the satellite.
%Input:
%   in_sun_pos - A 3xN matrix with the satellite positions. Positions where the
%                the satellite is in shadow, are marked as [0 0 0]'
%   time       - A vector with the time(in seconds) corresponding to the positions
%Output:
%   power - A vector with the generated power at each timestep
```

```
%Author(s): Kjell Magne Fauske, Daniel Berg
global s;
N=length(time);
E=zeros(1, length(time));
time=time*1/(3600*24);
disp('rotating vectors...');
rotatedsun=zeros(3,N);
for i =1:N
    T=time(i);
    sun=findSunDirection(T);
    rotatedsun(:,i)=rotateToBody(in_sun_pos(:,i), sun);
end
disp('Calculating energy...');
tic
for i = 1:N;
    if in_sun_pos(:,i) ~= [0 0 0]'
        s=rotatedsun(:,i);
        E(i)=quadl(@CalcPower, 0, 2*pi,1E-3); %use a numerical integration method.
    end
end
toc
disp('...done');
power=E;
```

CalcPower.m

```
function y=CalcPower(x)
%Calculate the power generated at a single timestep. Used by a numerical
%integration method.
%Author(s): Daniel Berg, Kjell Magne Fauske
global s; %sun vector in the orbit frame
global I_in;
global A0; global eta0;
global refraction_index;
n2=refraction_index;
nz=[0 0 1]';
N=length(x);
theta1=zeros(1,N);
theta2=zeros(1,N);
theta3=zeros(1,N);
theta4=zeros(1,N);
for i=1:length(x)
    alfa=x(i);
    nx=[cos(alfa) sin(alfa) 0]';
    ny=[sin(alfa) cos(alfa) 0]';
    theta1(i)=acos(nx'*s);
    theta2(i)=acos(ny'*s);
    theta3(i) = pi-theta1(i);
    theta4(i) = pi-theta2(i);
    %theta3(i)=acos((-nx)'*s);
    %theta4(i)=acos((-ny)'*s);
end
theta0=acos(nz'*s);

dt=1;
f=1;
k1=A0*I_in*dt/(2*pi)*eta0/eta2(0,n2);
```

```
y=k1.*(eta2(theta0,n2).*cos(theta0)+eta2(theta1,n2).*cos(theta1)
 +eta2(theta2,n2).*cos(theta2)+eta2(theta3,n2).*cos(theta3)
 +eta2(theta4,n2).*cos(theta4));
```